Master Thesis Notes

# Design of a Vibration Isolator:

Source: NPTEL “Vibration Control” Lectures Dr. S.P. Harsha

When designing vibration isolators, a few fundamental rules of thumb must be followed to ensure effective vibration isolation. These rules are:

1. **Isolator Stiffness (Static Stiffness):**

* The static stiffness of the isolator must be chosen such that the **highest mounting resonance** occurs at a frequency far below the **lowest exciting frequency**.
* This ensures that the isolator effectively reduces vibrations without amplifying any frequencies.

**2. Foundation Stiffness:**

* The mounting positions on the foundation should be as stiff as possible.
* A stiff foundation minimizes the risk of transferring vibrations from the isolator to the structure or vice versa.

**3. Coupling Stiffness:**

* The points at which the machine is coupled to the isolators should also be as stiff as possible.
* This ensures that the isolators can function properly without being affected by weak couplings.

**4. Internal Anti-Resonance:**

* The isolator should be designed so that its first internal anti-resonance frequency is well above the highest frequency of excitation.
* While effective, this is often challenging in practice. If this condition cannot be met, alternative rules (below) should be followed.

**5. Internal Resonances:**

* The isolator must be designed so that its internal resonances do not coincide with the strong components of the excitation spectrum.

**6. Foundation Resonances:**

* The isolator must also ensure that its anti-resonance frequencies do not coincide with the resonance frequencies of the foundation.

**Practical Constraints:**

In addition to the rules mentioned above, practical considerations such as geometric constraints, material strength, and stability concerns must be addressed.

**Example Use Case: Vibration Isolation in an Industrial Machine:**

**Scenario:** Consider a high-speed compressor installed in an industrial plant. The compressor generates vibrations at frequencies ranging from **20 Hz to 100 Hz**, and these vibrations can affect nearby equipment and structural components.

**Solution:**

1. **Choose a Soft Isolator:**
   * Select isolators with low static stiffness to ensure that the highest mounting resonance is below 20 Hz (the lowest excitation frequency).

Explanation:  
  
**Static stiffness** refers to the stiffness of the isolator material or structure under static (non-vibratory) loads. It is defined as:

Where:

* k: Static stiffness (N/m)
* F: Applied static load (force) (N)
* Δ: Resulting static deflection (m)

This is Why **Low Static Stiffness is Important** for Vibration Isolation

1. **Natural Frequency of the Isolator**: The natural frequency of an isolator is given by:
   * k: Stiffness of the isolator (N/m)
   * m: Mass of the supported machine (kg)

A **lower stiffness** (k) reduces the natural frequency (fn​) of the isolator. This helps to:

* + Shift the isolator's natural frequency below the operating frequency of the machine.
  + Ensure that the isolator achieves effective vibration isolation by operating in the isolation region (where excitation frequency > ​).

1. **Isolation Effectiveness**:
   * To isolate vibrations effectively, the isolator's natural frequency (​) must be much lower than the operating frequencies of the vibration source.
   * Low static stiffness helps achieve this by lowering​​.
2. **Deflection Considerations**:
   * While low stiffness improves isolation, it also leads to greater static deflection (Δ).
   * This requires balancing stiffness to avoid excessive deflection that could destabilize the machine.

**Example: Choosing Low Static Stiffness**

Suppose:

* A machine weighs 100 kg (m=100 kg).
* The operating frequency of the machine is 50 Hz.

To achieve effective isolation:

* The natural frequency of the isolator (​) should be much lower than 50 Hz, for suppose its consider the frequency to be **10 Hz**.
* Using the formula for ​​:

Substituting =10 Hz and m=100 kg:

A stiffness value of around **39,478 N/m** would give the required natural frequency of 10 Hz.

**Practical Implications of Low Stiffness**

1. **Improved Isolation**:
   * Vibrations at higher frequencies (e.g., 50 Hz and above) will be attenuated effectively.
2. **Static Deflection**:
   * A lower stiffness will result in greater deflection under the machine's weight:

For m = 100 kg and g = 9.81 m/s2

This deflection is acceptable in many cases but should be considered in the design.

1. **Stiffen the Foundation:**
   * Reinforce the floor slab beneath the compressor with steel plates or concrete to increase its stiffness.
2. **Ensure Proper Coupling:**
   * Use rigid bolts and plates to securely connect the isolators to the compressor base and the foundation.
3. **Avoid Internal Resonance Issues:**

Ensure that the isolator material and design do not amplify frequencies between 20 Hz and 100 Hz.

What does it mean?

* + Internal resonance occurs when the natural frequencies of the isolator’s components (such as rubber, springs, or internal structures) align with the excitation frequencies.
  + When this happens, vibrations are amplified instead of dampened, reducing the isolator's effectiveness.

**How to avoid internal resonances?**

* **Material Selection**: Use isolator materials with inherent damping properties (like viscoelastic materials) to suppress internal resonances.
* **Design Geometry**: Ensure that the dimensions, mass, and stiffness of the isolator are designed so that the isolator's natural frequencies lie outside the excitation frequency range. For example, if the excitation range is 20 Hz to 100 Hz, the isolator's natural frequencies should be below 20 Hz or well above 100 Hz.
* **Simulation and Testing**: Perform finite element analysis (FEA) or experimental modal analysis to identify the natural frequencies of the isolator. Adjust design parameters to shift these frequencies.

1. **Check Anti-Resonance Frequencies:**
   * Test the isolator system to ensure that its anti-resonance frequencies are well above 100 Hz (the highest frequency of interest).

**What is Anti-Resonance?**

* Anti-resonance is a frequency at which the response of a system is minimal due to destructive interference of vibrational energy.
* In vibration isolation, this refers to the frequency at which the vibration transmission to the foundation is nearly zero.

**Why is Anti-Resonance Important?**

* If the anti-resonance frequencies of the isolator align with the foundation's resonance frequencies, the vibrations can still amplify in the foundation, defeating the purpose of isolation.

**How to Avoid Anti-Resonance Issues?**

* **Avoid Matching with Foundation Resonances**: The foundation's natural frequencies should not overlap with the isolator's anti-resonance frequencies. This is done by analyzing the foundation's vibrational characteristics.
* **Tuning the Isolator**: Modify the isolator's design parameters (mass, stiffness, damping) to adjust the anti-resonance frequencies.

**Outcome:** This setup reduces the transfer of vibrations from the compressor to the surrounding structure and minimizes the risk of resonance amplification.

# State of Art Optimizing Techniques for Designing the Folded Beam Structures.

**1. Genetic Algorithms (GAs):** GAs are evolutionary algorithms that simulate natural selection processes to find optimal solutions. In the context of folded beam structures, GAs have been employed to optimize design parameters for improved performance. For instance, a two-stage genetic algorithm was proposed for beam–slab structure optimization, focusing on layout and component dimensions to achieve optimal designs.

[MDPI](https://www.mdpi.com/2075-5309/14/9/2932?utm_source=chatgpt.com)

**2. Bi-Directional Evolutionary Structural Optimization (BESO):** BESO iteratively removes inefficient material and adds material to high-stress regions, refining the structure's topology. This method has been applied to enhance the mechanical properties of beams, leading to designs with controlled deflections and improved performance.

[Nature](https://www.nature.com/articles/s41598-023-33946-x?utm_source=chatgpt.com)

**3. Principal Stress Line Method:** This technique involves designing beam structures based on principal stress lines, ensuring that material is placed along paths of maximum stress. It considers size, topology, and shape optimization simultaneously, resulting in structures with minimal compliance and maximum stiffness.

[UT Web](https://utw10945.utweb.utexas.edu/Manuscripts/2010/2010-56-Li.pdf?utm_source=chatgpt.com)

**4. Variational Asymptotic Method (VAM):** VAM is used for dimensional reduction in structural problems, such as beams, plates, and shells, to find stresses and strains based on small parameters. It's applicable in both macro and micro mechanics, aiding in the design and analysis of composite materials and multifunctional structures.

[Wikipedia](https://en.wikipedia.org/wiki/Variational_asymptotic_method?utm_source=chatgpt.com)

**5. Isogeometric Analysis (IGA):** IGA integrates finite element analysis (FEA) with computer-aided design (CAD) tools, allowing for precise geometry representation and analysis. It's particularly effective in optimizing three-dimensional curved beam structures for maximal fundamental frequency, enhancing their dynamic performance.

[arXiv](https://arxiv.org/abs/2101.09566?utm_source=chatgpt.com)

**6. Topology Optimization with Beam Element Recognition:** This method focuses on explicit topology optimization by recognizing beam elements within a structure. It adjusts node locations, element lengths, orientations, and sizes to minimize compliance, leading to efficient beam assemblies.

[arXiv](https://arxiv.org/abs/2103.08347?utm_source=chatgpt.com)

**7. Multi-Objective Optimization:** In certain applications, such as the design of multi-resonant folded beam piezoelectric energy harvesters, optimization involves balancing multiple objectives. Techniques like global evolutionary algorithms are used to explore the design space, followed by local optimization methods to fine-tune the design for optimal performance.

[IEEE Xplore](https://ieeexplore.ieee.org/abstract/document/9152684?utm_source=chatgpt.com)

**Based on the seven methods listed, Multi-Objective Optimization** stands out as one of the most recent and promising techniques, particularly for designing complex folded beam structures, such as in MEMS devices or vibration isolation platforms.

Here’s why **Multi-Objective Optimization** is noteworthy:

**Key Reasons:**

1. **Balancing Multiple Objectives:**
   * Unlike single-objective optimization (e.g., minimizing natural frequency or displacement), multi-objective optimization simultaneously considers conflicting objectives, such as:
     + Minimizing weight while maximizing stiffness.
     + Maximizing vibration isolation while maintaining structural integrity.
   * This holistic approach produces a **Pareto front**, allowing designers to choose the best trade-off among competing objectives.
2. **Recent Research and Applications:**
   * For instance, a **2020 study on multi-resonant folded beam piezoelectric energy harvesters** used **global evolutionary algorithms** followed by local optimization for precise tuning. This approach achieved **higher energy harvesting efficiency** and **improved dynamic performance**. ([IEEE Xplore](https://ieeexplore.ieee.org/abstract/document/9152684))
   * This technique is often paired with advanced numerical methods, such as Finite Element Analysis (FEA), to ensure realistic designs.
3. **Integration with Modern Tools:**
   * Multi-objective optimization is widely integrated into platforms like **MATLAB**, **ANSYS**, or **COMSOL Multiphysics**, making it accessible for real-world engineering problems.
4. **Flexibility Across Applications:**
   * It adapts to diverse applications, from **vibration isolation platforms** to **energy harvesting devices** and **lightweight aerospace structures**.

**Which Other Method is Promising?**

If you're looking for **cutting-edge advancements in geometry-specific design**, **Isogeometric Analysis (IGA)** is another powerful technique. While slightly older than multi-objective optimization, its integration of CAD and FEA yields **outstanding precision**, particularly in optimizing 3D curved structures and highly complex beam geometries.

**Final Recommendation:**

For optimizing folded beams, start with **Multi-Objective Optimization** for its **versatility**, **recent success in research**, and ability to handle trade-offs. Use **IGA** as a complementary tool if the folded beam geometry becomes highly intricate and demands precision modeling.

For Isometric Analysis recommended resources:

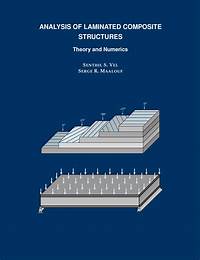
* 1. Video : Thomas J. R. Hughes, Isogeometric Analysis: Mathematical and Engineering Perspectives.
  2. TU Wien : Isogeometric Analysis course

## Specific Research Papers for the Optimization Techniques for Designing the Folded Beams

While specific research on multi-objective optimization methods for designing folded beams in vibration isolation is limited, several studies explore related optimization techniques for beam structures and vibration isolation systems. Here are some relevant papers:

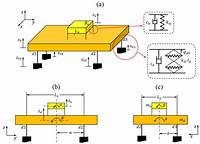
**Multi-objective optimization of laminated composite beam structures**  
This study employs the finite element method to analyze free vibration and uses multi-objective optimization to design laminated composite beams, aiming to minimize weight and maximize natural frequency.

[Delft University of Technology Research](https://research.tudelft.nl/files/11986319/Multi_objective_optimization_of_laminated_composite_beam.pdf?utm_source=chatgpt.com)



**Structural optimization of linear vibration isolation systems**  
This article addresses the structural optimization of linear vibration isolation systems under various actions, providing insights into optimizing structures for vibration isolation.

[ResearchGate](https://www.researchgate.net/publication/351109028_STRUCTURAL_OPTIMIZATION_OF_LINEAR_VIBRATION_ISOLATION_SYSTEMS?utm_source=chatgpt.com)



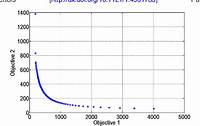
**Multi-objective optimization of elastic beams for noise reduction**  
This paper presents a study on the multi-objective optimization of elastic beams, focusing on minimizing weight and radiated sound power, which is relevant to vibration isolation applications.

[ResearchGate](https://www.researchgate.net/publication/318886896_Multi-Objective_Optimization_of_Elastic_Beams_for_Noise_Reduction?utm_source=chatgpt.com)



**Multi-objective optimization of acoustic black hole vibration absorbers**  
This research investigates the use of power law tapers to achieve the acoustic black hole effect for vibration reduction, offering insights into advanced vibration isolation techniques.

[AIP Publishing](https://pubs.aip.org/asa/jasa/article/140/3/EL227/649613/Multi-objective-optimization-of-acoustic-black?utm_source=chatgpt.com)



**Gradient zigzag metamaterial beams as broadband vibration isolators**  
This study proposes gradient zigzag metamaterial beams with flat bands within a broad frequency range, demonstrating their effectiveness as broadband vibration isolators for beam-like structures.

[arXiv](https://arxiv.org/abs/2110.09071?utm_source=chatgpt.com)

# Initial Design and Modelling for a Folded Beam Structure

In the view of vibration isolation which can isolate the unnecessary vibrations from the random orbital tool to the robotic arm for this purpose a need of a compact cost effective functing model is essential. Since the current system is vibrating in high frequency region ( 130 to approx. 200) Hz so there should be some platform which could isolate those high frequencies that could effect the moment of the robotic arm. For this the propose Isolator in most of the literature are used to isolate in the low frequency domain or else they are using the active complient methods to minimize the vibrations but in the currency study we are more focused to make a passive low cost effective isolator and one of the mechanisms is to use a **“Folded beam”** structure as a Vibration isolation platform. Some of the profound works on this domain are done by these following papers:

Since the review of the con

1. Modeling, design and analysis of low frequency platform for attenuating micro-vibration in spacecraft by D. Kamesh et al.,2010.
2. Experimental research on a vibration isolation platform for momentum wheel assembly by Weiyong Zhou et al.,2013.
3. Performance Analysis of a Flywheel Microvibration Isolation Platform for Spacecraft by Zhanji Wei et al.,2015.
4. Design and Modeling of Micro-vibration Isolation for Spacecraft Flywheel by Oing Luo et al.,2020.
5. Passive vibration isolation of reaction wheel disturbances using a low frequency flexible space platform by D. Kamesh et al.,2012.

A diagram of a flexible platform

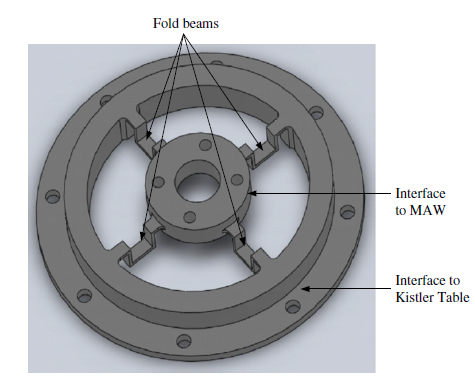
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1. Nonlinear static and dynamic response of a megastructure exhibiting quasi‑zero‑stiffness characteristics for vibration control an experimental validation Srajan Dalela et al., 2024 latest.
2. Design And Analysis Of Flexible Beam Platform As Vibration Isolator For Space Applications PHD Thesis by D. Kamesh. IISC Banglore, 2016.
3. Wei, Zhanji, et al. "Modeling and analysis of a flywheel microvibration isolation system for spacecrafts." Advances in Space Research 55.2 (2015): 761-777.

## Initial Ideas

The Initial idea is to use a folded beam model to control the oscillations what was occurring while the tool is moving in a orbital fashion for this case using a folded beam in a particular arrangement like this could mitigate the vibrations along the radial directions.

A diagram of a maze

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A diagram of a plywood structure

Description automatically generated with medium confidence

This design could be considered to design the minize the radial forces using the folded beam in this configuration.

A drawing of a letter l

Description automatically generated with medium confidence

Also Thinking of including a slotted disc if possible at the bottom of the base to support the load of the tool and this could look something similar to this.



Some Further design ideas for the Constant force mechanisms:

* Compliance-based modeling and design of straight-axis/circular-axis flexible hinges with small out-of-plane deformations by N. Lobontiu et al. 2014

A diagram of a mechanical device

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* Hou, Chia-Wen, and Chao-Chieh Lan. "Functional joint mechanisms with constant-torque outputs." Mechanism and Machine Theory 62 (2013): 166-181.
* Or Using a constant Force Spring for making the constant force mechanism something like including the constant force spring to the flexture mechanism.

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* Zhang, Xiaozhi, Guangwei Wang, and Qingsong Xu. "Design, analysis and testing of a new compliant compound constant-force mechanism." Actuators. Vol. 7. No. 4. MDPI, 2018. ( See once the way they presented the relation between the robot and the cfm mechanism) Zhang, Xiaozhi, Guangwei Wang, and Qingsong Xu. "Design, analysis and testing of a new compliant compound constant-force mechanism." Actuators. Vol. 7. No. 4. MDPI, 2018.

A diagram of a diagram of a diagram

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* A diagram of a green object with text and numbers

  Description automatically generated**Very Imp paper!** Chen, Yi-Ho, and Chao-Chieh Lan. "An adjustable constant-force mechanism for adaptive end-effector operations." (2012): 031005.
* Wei, Yuzhang, and Qingsong Xu. "Design and testing of a new force-sensing cell microinjector based on small-stiffness compliant mechanism." IEEE/ASME Transactions on Mechatronics 26.2 (2020): 818-829.
* Wei, Yuzhang, and Qingsong Xu. "Design of a new passive end-effector based on constant-force mechanism for robotic polishing." Robotics and Computer-Integrated Manufacturing 74 (2022): 102278.

A diagram of a stress distribution model

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* Luo, Qing, Jun Wu, and Caizhi Fan. "Design and Modeling of Micro-vibration Isolation for Spacecraft Flywheel." 2020 3rd International Conference on Mechatronics, Robotics and Automation (ICMRA). IEEE, 2020.

A diagram of a machine

Description automatically generated

* Ding, Bingxiao, Xuan Li, and Yangmin Li. "FEA-based optimization and experimental verification of a typical flexure-based constant force module." Sensors and Actuators A: Physical 332 (2021): 113083.

Diagram of diagram of a function

Description automatically generated with medium confidence

* Chen, Qi, et al. "Design of compliant constant-output-force mechanisms **using topology optimization**." Engineering Optimization 55.12 (2023): 1997-2014.

A close-up of several lines

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* A diagram of a bug

  Description automatically generatedZhang, Chen, et al. "Design of a locust leg-like compliant constant-force mechanism supporting large-scale damage-free manipulation." Review of Scientific Instruments 94.11 (2023). **(Very Imp!!)**

A diagram of a structure

Description automatically generated

* Lan, Chao-Chieh, Jhe-Hong Wang, and Yi-Ho Chen. "A compliant constant-force mechanism for adaptive robot end-effector operations." 2010 IEEE International Conference on Robotics and Automation. IEEE, 2010.

A diagram of a curve

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* **Some of the Survey / Literature review Papers:**

1. **Wang, Piyu, and Qingsong Xu. "Design and modeling of constant-force mechanisms: A survey." Mechanism and Machine Theory 119 (2018): 1-21.**
2. **Ling, Jie, et al. "A survey on synthesis of compliant constant force/torque mechanisms." Mechanism and Machine Theory 176 (2022): 104970.**
3. **Shi, H. T., et al. "Vibration isolation methods in spacecraft: A review of current techniques." Advances in Space Research (2024).**

An Additional Paper to check regarding the Vibration Isolation if anything does not work out

Wang, Yanmiao, et al. "Full-band vibration isolation and energy absorption via cuttlebone-inspired lattice structures." International Journal of Mechanical Sciences 274 (2024): 109283.

# Mode shapes and Modal Analysis

Mode shapes are always orthogonal to one another.

Note: Important point to remember – in every modal analysis the first mode gives the easiest way the structure gets deformed with the application of load or mass i.e., the structures least stiffness in any direction gives the first modal frequency.

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10x of the stain deflection plot.

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# Radius of Gyration and Its significance in Beams

The **radius of gyration** is a geometric property that describes how an object's mass or area is distributed about a specific axis. It is often used in structural engineering, mechanics, and dynamics.

* **Definition:**

The radius of gyration, denoted as kkk, is the distance from the axis at which the entire area (or mass) of an object could be concentrated to produce the same moment of inertia as the original distribution.

Mathematically:

k=IA(for area)ork=Im(for mass)k = \sqrt{\frac{I}{A}} \quad \text{(for area)} \quad \text{or} \quad k = \sqrt{\frac{I}{m}} \quad \text{(for mass)}k=AI​​(for area)ork=mI​​(for mass)

Where:

* III: Moment of inertia about the axis.
* AAA: Cross-sectional area (for area-related problems, e.g., column buckling).
* mmm: Total mass (for mass-related problems, e.g., dynamics).
* **Significance:**

1. **Structural Engineering (Columns):**
   * The radius of gyration is used to evaluate the **buckling strength** of columns.
   * A higher radius of gyration indicates a more stable column since the load distribution is farther from the axis, reducing the risk of buckling.
   * It is used in the formula for slenderness ratio: Slenderness Ratio=Leffk\text{Slenderness Ratio} = \frac{L\_{\text{eff}}}{k}Slenderness Ratio=kLeff​​ Where LeffL\_{\text{eff}}Leff​ is the effective length of the column.
2. **Mass Distribution in Dynamics:**
   * In rotating systems, the radius of gyration provides insight into how mass is distributed relative to the axis of rotation.
   * It affects angular motion and rotational energy: Erot=12Iω2E\_{\text{rot}} = \frac{1}{2} I \omega^2Erot​=21​Iω2 A larger kkk means the mass is farther from the axis, increasing the moment of inertia and rotational resistance.
3. **Moment of Inertia Simplification:**
   * By knowing the radius of gyration, one can calculate the moment of inertia without detailed knowledge of mass or area distribution: I=mk2orI=Ak2I = m k^2 \quad \text{or} \quad I = A k^2I=mk2orI=Ak2
4. **Design Optimization:**
   * In structural and mechanical design, optimizing kkk can lead to lighter yet stronger or more stable designs.

* **Example:**

For a rectangular column of width bbb and height hhh:

* Moment of inertia about the centroidal axis: I=112bh3I = \frac{1}{12} b h^3I=121​bh3
* Radius of gyration about the same axis: k=IA=112bh3bh=h212=h12k = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{1}{12} b h^3}{b h}} = \sqrt{\frac{h^2}{12}} = \frac{h}{\sqrt{12}}k=AI​​=bh121​bh3​​=12h2​​=12​h​

The radius of gyration gives an intuitive measure of the distribution of the cross-sectional area or mass for stability or rotational performance analysis.

4o

* **You said:**

could you proride me with a real time example to better understand the actual intubation behind it

* **ChatGPT said:**

ChatGPT

Certainly! Here's a **real-world example** to help you understand the concept and significance of the **radius of gyration**:

* **Example: Skyscraper Columns and Buckling Stability**

Imagine you're designing the steel columns of a tall skyscraper. These columns must be able to support the weight of the building and resist buckling under compressive loads.

1. **Why Radius of Gyration Matters**:
   * When a column is under compressive load, it tends to buckle if it’s too slender.
   * The **radius of gyration** is a measure of how the material is distributed around its centroid, which directly influences its **buckling strength**.
2. **How It Works in Design**:
   * A **circular column** has a high radius of gyration because its cross-section is symmetric and material is spread far from the center. This makes it **less likely to buckle**.
   * A **rectangular column**, on the other hand, may have different radii of gyration along its width and height. If the height is much larger than the width, it is **prone to buckling** along the weaker (shorter radius of gyration) axis.
3. **Real-Life Example**:
   * Consider two steel columns:
     + **Column A**: A circular column with radius of gyration k=50 mmk = 50 \, \text{mm}k=50mm.
     + **Column B**: A rectangular column with k=30 mmk = 30 \, \text{mm}k=30mm along one axis and k=10 mmk = 10 \, \text{mm}k=10mm along the other.
   * If both columns are of the same material, length, and cross-sectional area:
     + **Column A** is more stable because its material is evenly distributed and has a higher kkk, resisting buckling more effectively.
     + **Column B**, with a smaller kkk, would buckle more easily under the same load, especially along the weaker axis.
4. **Design Implication**:
   * To increase stability, engineers either increase the cross-sectional size or optimize the shape to increase the radius of gyration.
   * For instance, hollow tubes or I-beams are commonly used because they have a high kkk, offering excellent resistance to buckling without excessive weight.

* **Summary of Intuition:**

The **radius of gyration** tells us how "spread out" the material is around the centroid. For structural elements:

* A **larger radius of gyration** makes the structure more stable and resistant to buckling.
* Designers use shapes like tubes, wide flanges, or composite materials to maximize kkk, balancing weight and strength.

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# Euler Bernoulli beam bending theory

Important Vidoes and lectues to refer:

* <https://youtu.be/GHjopp47vvQ?si=FOQz1CStsX6Q26Sd>
* <https://youtu.be/S7zYcRtLMBc?si=x9b2FFtmW7sTMWro>
* <https://youtu.be/cHiFQ-cESkg?si=-IlkxWCujgIx6hwW>

A diagram of a shuttle

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# Frequency Response Analysis

A diagram of a weld

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